Tutorial III

Mathematics for economists - ENSL Premaster

October 13, 2022

Exercise 1.

For each of the following functions, compute $\sup_I f$ and $\inf_I f$. State if these bounds are reached, and if so, where.

- $f(x) = x(1-x)$ defined on $I = [0,1]$.
- $f(x) = 1 - e^{-x}$ defined on $I = [0, +\infty[$
- $f(x) = 3x^4 - 4x^3 + 6x^2 - 12x + 1$ defined on $I = \mathbb{R}$.
- $f(x) = \frac{1}{\sqrt{x^2-x+1}}$ defined on $I = [0,1]$.

Exercise 2.

Let $f \in \mathcal{F}(\mathbb{R}, \mathbb{R})$ be a continuous function such that $\lim_{+\infty} f(x) = \lim_{-\infty} f(x) = +\infty$. Show that f has a global minimum.

Exercise 3.

Let $f \in \mathcal{F}(\mathbb{R}_+, \mathbb{R})$ be a continuous function such that $\lim_{+\infty} f(x) = a \in \mathbb{R}$.

1. Show that f is bounded.
2. Show that f has a global extremum (minimum or maximum).

Exercise 4.

Compute the partial derivatives of the following functions defined for all $(x, y) \in \mathbb{R}^2$.

1. - $\forall (x, y) \in \mathbb{R}^2,\ f(x, y) = \cos(x)\sin(y)$
   - $\forall (x, y) \in \mathbb{R}^2,\ f(x, y) = \sqrt{1 + x^2 + y^2}$
   - $\forall (x, y, z) \in \mathbb{R}^3,\ f(x, y, z) = e^{x^2+y^2}\log(1 + z^4)$
   - $\forall (x, y) \in \mathbb{R}^2,\ f(x, y) = x^3 + (x^2 - 6)y + x^2$
2. Find the local and global minima of the last function.